因果格林函数：

$$\left(i\hbar\frac{\partial}{\partial t}-\hat{H}(\vec{r})\right)G(\vec{r},t;\vec{r}',t')=\hbar\,\delta(\vec{r}-\vec{r}')\,\delta(t-t')$$

$$\psi(\vec{r},t)=\int d\vec{r}'\,dt'\,G(\vec{r},t;\vec{r}',t')\,\psi(\vec{r}',t')$$

S矩阵：$\hat{\psi}(t)=S(t,t')\hat{\psi}(t')$

相互作用表象下：$S(t,t')=T\,e^{-i\int_{t'}^{t}dt_1\hat{V}(t_1)}$

caused

零温下因果格林函数（time-ordered 格林函数）

场算符表示：
$$G(x,t;x',t')=-\frac{i}{\hbar}\frac{\langle\Psi_0|T\{\hat{\psi}_H(x,t)\hat{\psi}_H^{+}(x',t')\}|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}$$
$$\equiv-\frac{i}{\hbar}\langle T\,\hat{\psi}_H(x,t)\,\hat{\psi}_H^{+}(x',t')\rangle$$

要点1：$\hat{\psi}_H(x,t)$ 为 Heisenberg picture 中定义的场算符

$$\hat{\psi}_H(x,t)=e^{i(\hat{H}-\mu\hat{N})t}\,\psi(x)\,e^{-i(\hat{H}-\mu\hat{N})t}$$

要点2：$|\Psi_0\rangle$ 体系基态波函数 （玻色子：直积，费米子：slater 行列式）

绝热引入相互作用：无相互作用时体系波函数 $|\Phi_0\rangle$

$$|\Psi_0\rangle=S(0,-\infty)|\Phi_0\rangle,\quad\langle\Psi_0|=\langle\Phi_0|S(\infty,0)$$

$$G(x,t;x',t') = -\frac{i}{\hbar}\frac{\langle\Phi_0|T\{S(+\infty,-\infty)\hat{\Phi}_H(x,t)\hat{\Phi}_H^{\dagger}(x',t')\}|\Phi_0\rangle}{\langle\Phi_0|S(+\infty,-\infty)|\Phi_0\rangle}$$

要点3：物理含义：传播子 $G(x,t;x',t')$

x'处，t'时产生一个粒子，t时刻x处该粒子湮灭的几率幅

要点4：具体作用

$$\underline{\left(i\hbar\frac{\partial}{\partial t} + \hat{H}(x)\right) G(\vec{r},t;\vec{r}\,',t')}$$

关键是 $\boxed{i\hbar\frac{\partial}{\partial t} G(\vec{r},t;\vec{r}\,',t')}$

$$T(\hat{A}(t)\hat{B}(t')) = \Theta(t-t')\hat{A}(t)\hat{B}(t') \mp \Theta(t'-t)\hat{B}(t')\hat{A}(t)$$

（-为费米子，+为玻色子）

$$i\hbar\frac{\partial}{\partial t} G(\vec{r},t;\vec{r}\,',t') = i\hbar\frac{\partial}{\partial t}\langle\Theta(t-t')\hat{\Phi}_H(x,t)\hat{\Phi}_H^{\dagger}(x',t') \mp \Theta(t'-t)\hat{\Phi}_H(x',t')\hat{\Phi}_H^{\dagger}(x,t)\rangle$$

$$= \langle\underbrace{\delta(t-t')\hat{\Phi}_H(x,t)\hat{\Phi}_H^{\dagger}(x',t') + \delta(t-t')\hat{\Phi}_H(x',t')\hat{\Phi}_H(x,t)}\rangle$$

费米子 $\delta(t-t')\delta(x'-x)$

$$+\langle T\, \underbrace{i\hbar\frac{\partial}{\partial t}(\hat{\Phi}_H(x,t)}_{\hat{H}\hat{\Phi}_H(x,t)}\hat{\Phi}_H^{\dagger}(x',t'))\rangle$$

有限温：$G(x,t;x',t') = -\frac{i}{\hbar}\langle\langle T\,\underbrace{\hat{\Psi}_H(x,t)\,\hat{\Psi}_H^{+}(x',t')}_{\text{简写为}\hat{\Psi}_H}\rangle\rangle$

$$\langle\langle\cdots\rangle\rangle = \frac{tr(e^{-\beta(\hat{H}-\mu\hat{N})}\cdots)}{tr(e^{-\beta(\hat{H}-\mu\hat{N})})}$$

$tr(e^{-\beta(\hat{H}-\mu\hat{N})}) \to$ 配分函数

自由粒子格林函数：

①体系粒子间无相互作用

②体系有相互作用 ——→ 计算相互作用对格林函数的修正（$G^{(0)}, G^{(1)}\cdots$）

例子：$\hat{K} = \hat{H}_0 - \mu\hat{N} = \sum_{\vec{k}} \varepsilon_{\vec{k}}\, \hat{a}_{\vec{k}}^{+}\hat{a}_{\vec{k}}$

$\varepsilon_{\vec{k}} = W_{\vec{k}} - \mu$，比如电子 $\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m} - \mu$

体积归一化

$$\Psi(\vec{r}) = \frac{1}{\sqrt{V}}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}\,\hat{a}_{\vec{k}}(t)$$

Heisenberg 表象：
$$\begin{cases} \hat{a}_{\vec{k}}(t) = \hat{a}_{\vec{k}}\cdot e^{-i\varepsilon_{\vec{k}}t} \\ \hat{a}_{\vec{k}}^{+}(t) = \hat{a}_{\vec{k}}^{+}\cdot e^{i\varepsilon_{\vec{k}}t} \end{cases}$$

$$G^{(0)}(\vec{r},t;\vec{r}',t') = -\frac{i}{\hbar}\langle T\,\hat{\Psi}(\vec{r},t)\,\hat{\Psi}^{+}(\vec{r}',t')\rangle$$

$$= -\frac{i}{V\hbar}\sum_{\vec{k}\vec{k}'} e^{i(\vec{k}\cdot\vec{r}-\vec{k}'\cdot\vec{r}')}\langle T\,\underbrace{\hat{a}_{\vec{k}}(t)\,\hat{a}_{\vec{k}'}^{+}(t')}_{\delta_{\vec{k}\vec{k}'}}\rangle$$

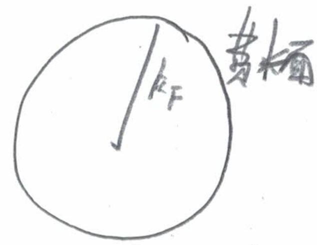
$$= \begin{cases} -\frac{i}{\hbar V}\sum_{\vec{k}} e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}\, e^{-i\varepsilon_{\vec{k}}(t-t')}\,\langle \underbrace{\hat{a}_{\vec{k}}\,\hat{a}_{\vec{k}}^{+}}_{1-f_{\vec{k}}}\rangle, & t>t' \\ -\frac{i}{\hbar V}\sum_{\vec{k}} e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}\, e^{-i\varepsilon_{\vec{k}}(t-t')}\,\langle \underbrace{\hat{a}_{\vec{k}}^{+}\,\hat{a}_{\vec{k}}}_{f_{\vec{k}}}\rangle, & t<t' \end{cases}$$

$\longrightarrow G^{(0)}(\vec{r}-\vec{r}', t-t')$

$$
\begin{cases}
G^{(0)}(\vec{k},\omega) = \dfrac{1-f_{\vec{k}}}{\omega-\varepsilon_{\vec{k}}+i0_+} + \dfrac{f_{\vec{k}}}{\omega-\varepsilon_{\vec{k}}-i0_+} & (费米子) \\[2ex]
G^{(0)}(\vec{k},\omega) = \dfrac{1+n_{\vec{k}}}{\omega-\varepsilon_{\vec{k}}+i0_+} - \dfrac{n_{\vec{k}}}{\omega-\varepsilon_{\vec{k}}-i0_+} & (玻色子)
\end{cases}
$$

$T=0$时，零温格林函数

费米子：
$$
\begin{cases}
f_{\vec{k}} = \Theta(k_F-|\vec{k}|) \\
1-f_{\vec{k}} = \Theta(|\vec{k}|-k_F)
\end{cases}
$$



Abrikosov

因果格林函数：传播子

$$
G^{(0)}(\vec{k},\omega) = \frac{\Theta(|\vec{k}|-k_F)}{\omega-\varepsilon_{\vec{k}}+i0_+} + \frac{\Theta(k_F-|\vec{k}|)}{\omega-\varepsilon_{\vec{k}}-i0_+} \quad (费米子)
$$

$$
G^{(0)}(\vec{k},\omega) = \frac{1}{\omega-\varepsilon_{\vec{k}}+i0_+} \quad (玻色子)
$$

空间平移不变：$G(\vec{r}-\vec{r}'; t-t')$

$$
G(\vec{k}, t-t') = -i\langle T\, \hat{a}_{\vec{k}}(t)\, \hat{a}^{+}_{\vec{k}}(t')\rangle \qquad 动量空间
$$

$\rightsquigarrow$ 无相互作用（如散射驱动） 粒子一直在k上

$G^{(0)}(\vec{r},t;\vec{r}',t')$ 为 $G^{(0)}(\vec{r}-\vec{r}',t-t')$,只与 $t-t'$, $\vec{r}-\vec{r}'$ 有关

将 $G^{(0)}(\vec{r}-\vec{r}',t-t') \longrightarrow G^{(0)}(\vec{k},\omega)$

$$G^{(0)}(\vec{r}-\vec{r}',t-t')=\int\frac{d\vec{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\underbrace{G(\vec{k},\omega)}_{\text{动量频率空间}}e^{i\vec{k}\cdot(\vec{r}-\vec{r}')-i\omega(t-t')}$$

$$\to G^{(0)}(\vec{k},\omega)=\int d\vec{r}\,dt\,\underbrace{G^{(0)}(\vec{r},t)}_{\text{分段函数}}e^{-i\vec{k}\cdot\vec{r}+i\omega t}$$

分段计算: $t>0$ 时

$-i0_+$

$$G^{(0)}(\vec{k},\omega)=\int_0^{\infty}dt\,(-i)\,e^{-i(\varepsilon_{\vec{k}}-\omega)t}(1-f_{\vec{k}})$$

$$+\int_{-\infty}^{0}dt\,(i)\,e^{-i(\varepsilon_{\vec{k}}-\omega)t}f_{\vec{k}}$$

$+i0_+$

$$=\frac{1-f_{\vec{k}}}{\omega-\varepsilon_{\vec{k}}+i0_+}+\frac{f_{\vec{k}}}{\omega-\varepsilon_{\vec{k}}+i0_+}$$

作业: 将 $G(\vec{k},\omega)$ 逆变换为 $G(\vec{r},t)$,回到分段函数形式

定义 $G^>$, $G^<$ 格林函数

$$G^>(\vec{r}, t-t') = -\frac{i}{\hbar}\langle \hat{a}_{\vec{r}}(t)\hat{a}^+_{\vec{r}}(t')\rangle$$

$$G^<(\vec{r}, t-t') = \frac{i}{\hbar}\langle \hat{a}^+_{\vec{r}}(t')\hat{a}_{\vec{r}}(t)\rangle$$

费米子

则 $\underbrace{G(\vec{r}, t-t')}_{\text{因果格林函数}} = \Theta(t-t')\,G^>(\vec{r}, t-t') + \Theta(t'-t)\,G^<(\vec{r}, t-t')$

$$G^{\gtrless}(\vec{r}, t-t') = \int \frac{d\omega}{2\pi} e^{-i\omega t}\, G^{\gtrless}(\vec{r}, \omega)$$